

PUTNAM PRACTICE SET 32

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Problem 1. Let M be an even positive integer. Show that for each positive integer n , the number

$$M^{M^{M^n}} + M^{M^n} + M^n - 1$$

is a not prime.

Problem 2. Let A , B and C be noncollinear points in the plane with integer coordinates such that also the three distances between the points (AB , BC and CA) are integer numbers. What is the smallest possible value for AB ?

Problem 3. Find all pairs of polynomials $P(x)$ and $Q(x)$ with the property that

$$P(x)Q(x+1) - P(x+1)Q(x) = 1.$$

Problem 4. Let $n \in \mathbb{N}$ and let $A \in M_{n,n}(\mathbb{R})$. For each $k \in \mathbb{N}$, we denote by $A^{[k]}$ the n -by- n matrix whose entries are the k -th powers of the corresponding entries in A . If

$$A^{[k]} = A^k \text{ for each } 1 \leq k \leq n+1,$$

then $A^{[k]} = A^k$ for all $k \geq 1$.